L¹ Embeddability, Complexity and Multicommodity Flows

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Abstract

A finite metric $d$ (or more properly semimetric) is $L¹$-embeddable if it can be expressed as a non-negative combination of cut metrics. When $d$ is rational this is equivalent to a multiple of $d$ being isometrically embeddable in a hypercube of suitably high dimension. The cone of $L¹$-embeddable metrics is called the Hamming cone. In this talk we provide a unified setting for describing a number of results related to $L¹$-embeddability. We collect and describe results on the facial structure of the Hamming cone and the complexity of testing $L¹$-embeddability of a metric. We describe the role of such metrics in a number of areas including multicommodity flows, combinatorial optimization and measure theory. In particular we show how the specializations of the so-called Japanese Theorem, for the feasibility of multicommodity flows, to a Ford-Fulkerson type theorem depend critically on the $L¹$-embeddability of certain small metrics. Finally we give some results on the facial structure of cones generated by subsets of cut metrics.