Optimal Enclosure Problems

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Abstract

We consider the following "fence enclosure" problem: Given a set S of n points in the plane with weights $w_i \geq 0$, we wish to enclose a subset of the points with a fence (a simple closed curve) so as to maximize the "value" of the enclosure. The value of the enclosure is defined to be the sum of the weights of the enclosed points minus the cost of the fence. We assume that fence costs \$c per unit length.

We consider various versions of the problem, such as allowing S to consist of points and/or simple polygons. We assume that no fence is permitted to pass through any polygon of S. Other versions of the problems are obtained by restricting the total amount of fence available and also allowing the enclosure to consist of up to K connected components. There may also be constraints on the allowable shapes of the fence.

We show that the problem in two dimensions for a bounded length fence is NP-complete by a reduction from Knapsack. Additionally we provide polynomial-time algorithms for many versions of the fence problem when an unrestricted amount of fence is available. For the one-dimensional case we provide linear-time algorithms for most versions of the problem, assuming the points are available sorted along the line. When the set S consists of points in the plane and the fence is unrestricted we solve the problem via an $O(n^3)$ time sweep-line algorithm. For the case just mentioned we give an alternative $O(n^4)$ time algorithm based on dynamic programming which easily generalizes to higher dimensions (in time $O(n^{2d-1})$). In two dimensions the dynamic programming approach yields polynomial-time algorithms when we allow the set S to consist of simple polygons or when $K > 1$ components are permitted (polynomial for constant K). We suspect that if K is part of the input, then the multiple fence problem may be NP-complete. By "critical placement" arguments we can solve the version in which the shape of the fence is fixed in polynomial time.