Planar Conflict Resolution for Air Traffic Control*

Yui-Bin Chen†  Mansi Hsieh†  Alfred Inselberg‡  Homer Q. Lee§

April 11, 1990

Keywords: Motion Planning, Moving Objects, Conflict Detection and Resolution, Air Traffic Control, and Relative Coordinates.

1 Introduction and Problem Definition

Motion planning with moving objects is computationally harder than with stationary objects. Here such a problem arising in Air Traffic Control (ATC) is studied. The goal of ATC is to direct aircraft safely to their destinations while minimally interfering with their intended trajectories. This involves maintaining a minimum separation distance $d$ (usually 5 nautical miles(n.m.)) between aircraft (considered as moving points), detecting conflicts (when the distance between aircraft is less than $d$), and resolving the conflicts according to certain maneuvering priorities and constraints.

Conflict Resolution being an instance of the Asteroid Avoidance Problem is an NP-hard problem. Resolution methods have been proposed which resolve the conflicts subset by subset, leading to backtracking and very high complexities. Such schemes can also lead to worse conflicts than the ones they resolved and pose some fundamental difficulties in program proving.

In this paper, we provide a new approach using a normalized relative coordinate model to both detect and resolve the conflicts. For each maneuvered aircraft, we represent it as three pseudo-aircraft indicating the status

---

*Research supported by NASA contract NCC-2-587.
†Department of Computer Science, University of Southern California, Los Angeles, CA 90089-0782.
‡IBM Scientific Center, 2525 Colorado Avenue, Santa Monica, CA 90404 (aiisreal@ibm.com).
§NASA Ames Research Center, Moffett Field, CA 94035.
of that aircraft in different time periods (See Figure 1 and Table 1). For the multi-aircraft case, the resolutions obtained by the algorithm depend on the input sequence order. Here, heuristics are used to fix the order in the algorithm, whose time complexity is $O(N^2)$ without backtracking.

Even though the general problem is 3-dimensional in space, it is preferable, when possible, to resolve conflicts without altitude changes. For this reason the 2-dimensional case is studied first. The constraints for the evasive maneuvers are:

- maintain constant speed before, during and after the maneuver,
- execution of maneuver should be no earlier than some specified lead time,
- there is a maximum allowable turn angle,
- there is a maximum allowable deviation from the original track (offset), and
- upon completion, the aircraft returns to its original heading (see Figure 1).

The requirement for the lead time is due to the uncertainty in the radar tracking and also for avoiding unnecessary maneuvers in case the aircraft were going to turn anyway. Returning to the same heading facilitates maintaining course using directional radio navigation. It is difficult to control speed variations and for this reason constant speed maneuvers are preferred.

2 Method

2.1 Normalized Relative Coordinates

A moving circle with radius $d/2$ centered at each airplane is considered. Maintaining the minimum separation is equivalent to preventing these circles from intersecting (though they may touch). Starting with a pair of aircraft one circle is shrunk to a point (referred to as a moving point) and denoted by $AC_i$ and the other, for aircraft $AC_j$, is doubled in radius without loss in generality.

The information is next transformed to a normalized relative coordinate system, in the sense that the information about all circles and their relative velocities with respect to a specific aircraft is presented on the same diagram, see Figure 2.

2.2 Constraints

To resolve the conflict, first, the lead time constraint is considered by translating the position of $AC_j$ in the negative X direction. To avoid the conflict,
\( \vec{V}_{ij} \) needs to be outside the conflict range \((\theta_{j1}, \theta_{j2})\) (see Figure 3). This may be accomplished by a rotation equivalent to changing the direction of AC_i and preserving the magnitude \( |\vec{V}_i| \), so as to satisfy the equal speed requirement.

When the maneuver starts, the course of AC_i is updated to that of pseudo-aircraft AC_{i'} (see Figure 1). Suppose that \( \varphi_i \) is chosen as the turn angle of AC_i causing \( \vec{V}_{ij} \) to rotate by some angle \( \psi_{ij} \), then

\[
\varphi_i = \angle \vec{V}_{i'} - \angle \vec{V}_i \iff \psi_{ij} = \angle \vec{V}_{i'j} - \angle \vec{V}_{ij} \quad (\angle \vec{V} \text{ means \"the angle of } \vec{V}\").
\]

The strategy for satisfying the offset constraint is shown in Figure 4.

In effect, as shown in Table 1, AC_i successively "spawns" 2 pseudo-aircraft, AC_{i'} and AC_{i''} (indicating the portion after the maneuver), for specified time periods.

**Table 1. The Information of Three Pseudo-Aircraft**

<table>
<thead>
<tr>
<th>Aircraft ID</th>
<th>Speed</th>
<th>Heading</th>
<th>Existing Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC_i</td>
<td>(</td>
<td>\vec{V}_i</td>
<td>)</td>
</tr>
<tr>
<td>AC_{i'}</td>
<td>(</td>
<td>\vec{V}_i</td>
<td>)</td>
</tr>
<tr>
<td>AC_{i''}</td>
<td>(</td>
<td>\vec{V}_i</td>
<td>)</td>
</tr>
</tbody>
</table>

\( T_{i1} \): the turn time of AC_i; lead time before the first conflict.
\( T_{i2} \): the earliest turn-back time satisfying all of the constraints.

### 2.3 Multi-Aircraft Resolution

For the multiple aircraft situations, the two-aircraft resolution algorithm is extended, so that the moving point avoids all other circles (including the ones due to the pseudo-aircraft), by using the normalized relative coordinate system. And, then, all of the aircraft are resolved one by one, using this extended model.

An example on realistic data provided by the FAA is shown in Figure 5. The ordering of the input sequence is pre-decided by a heuristic resulting in an \( O(N^2) \) algorithm. Hence the time complexity of the whole algorithm is bounded by \( O(N^2) \) and the space complexity is \( O(N) \).
Figure 1. Notation for Aircraft $i$ with Maneuver

Figure 2. Conflict Angle Range of $\bar{V}_{ij}$

Figure 3. Conflict-free Turning Angle Range

Figure 4. Conflict-free Turn-back Period

Figure 5(a). 5 Aircraft Unresolved

Figure 5(b). Resolution of 5 Aircraft