Algorithms for Ham Sandwich Cuts

William Steiger

Department of Computer Science
Rutgers University

Given disjoint sets $P_1, P_2, \ldots, P_d$ in $R^d$ with $n$ points in total, a ham-sandwich cut is a hyperplane that simultaneously bisects the $P_i$. Algorithms for finding ham-sandwich cuts in every dimension $d > 1$ will be described. When $d = 2$, the algorithm is optimal, having complexity $O(n)$. For dimension $d > 2$, the bound on the running time proportional to the worst-case time needed for constructing a level in an arrangement of $n$ hyperplanes in dimension $d - 1$. This, in turn, is related to the number of $k$-sets in $R^{d-1}$. With the currently known estimates, the complexity is close to $O(n^{3/2})$ for $d = 3$, roughly $O(n^{8/3})$ for $d = 4$ and $O(n^{d-1-a(d)})$ for some $a(d) > 0$ (going to zero as $d$ increases) for larger $d$. It is surprising that the complexity of finding a ham-sandwich cut is linear in $R^3$ and $R^4$ if the sets are suitably separated.