On Approximate Range Searching – or – Get in Shape; Round is a Good Choice

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Abstract

Range searching is among the most fundamental and well-studied problems in computational geometry. Given an n-element point set in $\mathbb{R}^d$, the problem is to preprocess the points so that the total weight, or generally the semigroup sum, of the points lying within a given query range η can be determined quickly. Years of research have resulted in nearly matching upper and lower bounds for many formulations of this problem. In approximate range searching, the user provides an $\varepsilon > 0$, and the problem is to determine the semigroup sum of all the points lying within η and additionally any subset of points lying within distance $\varepsilon \cdot diam(\eta)$ of η's boundary.

For many formulations the complexity of exact range searching is rather insensitive to semigroup properties and range properties. Recent research has indicated that matters are much different for approximate range searching. In approximate range searching the interaction of semigroup properties (such as idempotence) and range shape properties (such as smoothness) can have a significant impact on the complexity of answering queries. In this talk we will explore the differences between exact and approximate range searching, survey recent complexity results, present links to other retrieval problems, and discuss open problems and challenges for future research.