

Image Compression Terrain Simplification

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Abstract. Surface simplification is an important application in geographic information systems. The goal is to obtain a new surface that is combinatorially as simple as possible, while maintaining a prescribed degree of similarity with the original input surface.

In this paper, we propose a new terrain simplification algorithm which is based on known Digital Image Processing compression methods (e.g. *DCT*, wavelets compression) that was specially adjusted to fit Digital Elevation Models. DEM-images are terrains or elevation maps represented as gray scale images. We investigate the special nature of such terrain-images and design a unique pre-compression process which defines the parameters to guide the image compression. We perform a large-scale experiment comparing several terrain simplification methods and conclude that the new suggested algorithm (named *ICTS*¹) leads to significantly better compression results.

1 Introduction

Terrain models are commonly used to represent the surface of the earth or planets, as well as virtual worlds in games. The compression of such models is fundamental for a number of applications including storage, transmission and real-time visualization in navigation systems. The storage and transmission of high-resolution elevation information can consume considerable amounts of resources. The increased interest in GIS application, in particular, mapping the earth surface and real-time map representation, emphasize the need to develop efficient compression techniques for elevation maps.

Lossless compression methods [18, 21] often leads to compression ratio which is not high enough. Therefore in cases where some loss of information is allowed, and drastic compression ratio is needed (e.g. video, images audio), lossy compression is used (e.g. *JPEG* [12] for natural images). Lossy compression methods are mostly suitable when the distribution of elevation values is relatively flat. Elevation maps and terrains are similar in this sense to most natural images. In such scenes, high gradient values or discontinuities are rare and most content changes gradually across the domain. Examining the spatial frequency domain of elevation maps shows that the lower spatial frequency components contains more energy than the high frequency components, which often correspond to details and noises.

In this paper we assume the original surface is a grid based terrain (e.g. DEM, DTED). We choose to represent such terrain as a *2D* image where the pixel values are the height. We call this representation *elevation image*. Using elevation images enables the use of known digital image processing algorithms for compression (we use the term *terrain image* for a compressed elevation image).

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1.1 Motivation

We were motivated by several GIS application which deals with huge data sets of *DEMs*², which should be compressed in the most efficient and accurate way, an example for such application is NASA’s project of mapping Mars (MDIM project [19]). Other key applications which may benefit from the suggested method are navigation tools for mobile devices (e.g. GPS, smart-phone, PDA) these devices can hardly support triangulation and 3D visualization algorithms, as for limited computation power. Yet such devices often have basic image manipulation capabilities (zoom tilt) over standard image formats (i.e. *JPEG*), those capabilities can be used for implementing the suggested *ICTS* method on such devices. Other advantages of using standard image processing algorithms as terrain simplification and representation method, includes the improved runtime - DIP algorithms are extremely fast and can support huge data set as for the ‘local’ nature of the compression, moreover these algorithms can be implemented in dedicated hardware (GPU [7]) which can lead to a further runtime improvement.

Our contribution. This work demonstrates utilization of image compression for terrain simplification and representation. The suggested method (*ICTS*) is based on existing digital image processing method that was specially ‘tuned’ to fit the task of simplifying and representing terrain-images. We suggest a pre-compression stage which consider the terrain geometric properties. And show how *ICTS* can support geometric point location queries. We compare *ICTS* to the state-of-the-art image compression *JPEG2000* and to standard terrain simplification algorithms and conclude that the suggested *ICTS* method leads to better simplification results.

1.2 The paper structure

The rest of this paper is organized as follows; in section 2, we review previous work and several basic methods for terrain simplifications and terrain-image compression. Then, in section 3, we present our new *ICTS* terrain simplification method based on image compression algorithms. We discuss the importance of setting the parameters in order to convert a general compression algorithm into a practical terrain simplification method. The actual value of these parameters have a significant influence on the quality of the compression. In Section 4 we discuss the usability of compressed image-terrains as geometric surfaces. We demonstrate how terrain-image can support local queries such as: *point location*, *line of site(LOS)*, or *region of interest(ROI)* without decompressing all the image. In section 5 we report large scale experiment with *ICTS*, as well as other terrain simplification software packages. Finally, in section 6 we conclude this paper with a short discussion and some conclusions.

2 Related work

There are numerous papers dealing with terrain and surface simplification. Heckbert and Garland [10] surveyed general methods for simplifying 3D models (terrains are special case of 3D models). There has also been extensive work on many aspects specific to terrain simplification;

² Elevation maps may be stored as *DEM* (Digital Elevation Model) or *DTED* short for Digital Terrain Elevation Data that is a standard digital dataset, which is a uniform matrix of terrain elevation values providing basic quantitative data for systems and applications that require terrain elevation, slope, and/or surface roughness information.

most papers address terrain simplification using triangulation based representation and algorithms optimized to consider error norms such as maximum vertical distance and Hausdorff distance [6, 9, 22]. More recently several attempts to improve the terrain simplification accuracy and runtime results were made using semi-local triangulation [14, 15], and hybrid approach [24]. Little and Shi [17] (extending earlier work of Fowler and Little [4]), use linear features, along ridges and channels, to guide their triangulation algorithm with the goal of minimizing the root-mean-square (RMS) error in approximating a DEM with a triangulation.

On the other hand, terrains can be naturally represented by a 2D grid of heights (*DEM*). Such *DEM* terrain can be represented as a grayscale image, where each elevation sample is translated to a grayscale 'pixel' value. Few papers looked at such terrain-images from the digital image processing aspect; Franklin and Said [5] showed that the *progcode* image processing algorithm yields to efficient compression results over terrain images with respect to *RMS* error norm. Rane and Sapiro [21] investigated terrain-images lossless compression using the standard *JPEG-LS* [12]. Yea et al. [25] presented a detailed wavelets based compression for terrain-images which support elevation query mechanism allowing de-compressing only the parts of the terrain within an elevation range. Recently Owen and Grigg [20] demonstrated the use of *JPEG2000* for compressing and querying *DEMs*. Gortler and Hoppe [8] even define geometry images as a representation for closed 3D surface meshes.

In this paper we compare between these two main approaches; (i) Computational Geometry: triangulation based terrain simplification and (ii) Digital Image Processing: terrain-image compression. We compare the suggested new *ICTS* method with the following simplification methods: *QSlim* [6], *Terra* [9], *GcTin* [22], *JPEG2000* [13].

QSlim. *QSlim*, developed by Garland and Heckbert [6], is an algorithm designed for more general simplification of all types of surfaces and not just terrains. *QSlim* uses simple edge contraction to perform simplification, while using a quadric error measure for efficiency and for visual fidelity.

Terra. This algorithm, implemented by Garland [9], is based on a simple greedy insertion algorithm with some optimizations to make it run faster. The input is assumed to be a height field given by a regular grid of elevation data. It begins with a trivial triangulation of the domain and then iteratively adds vertices according to which input point has the greatest vertical error with respect to the approximating surface. Retriangulation is done using the Delaunay triangulation.

GcTin. *GcTin*, developed by Silva et al. [22], uses an advancing-front technique for simplification of digitized terrain models. The algorithm takes greedy cuts ("bites") out of a simple closed polygon that bounds a connected component of the yet-to-be triangulated region.

The method begins with a large polygon, bounding the whole extent of the terrain to be triangulated, and works its way inward, performing at each step one of three basic operations: ear cutting, greedy biting, and edge splitting. One of the main advantages of *GcTin* is that it requires very little memory beyond that for the input height array.

JPEG2000. *JPEG2000*, [13], is a new image coding system that uses state-of-the-art compression techniques based on wavelet technology. This image compression format is not yet commonly in use but has a compression advantage over *JPEG* [12] by roughly 20% on average. The compression gains over *JPEG* are attributed to the use of DWT and a more sophisticated entropy encoding scheme.

Clearly there other image compression methods, indeed as shown in this paper any image compression format can be thought as a terrain simplification method. We used *JPEG2000* as a point of reference as the most advance general image compression format available.

3 New Terrain Simplification Method (ICTS)

In this section we present our new Image Compression Terrain Simplification (*ICTS*) method that is based on existing DIP formats. We first cover the general notion of image compression using spatial frequency domain and investigate the difference between a (natural) image compression method, and a terrain-image compression method in terms of error norms. Lossy image compression methods are mostly suitable when the distribution of elevation values is relatively flat. Elevation maps and terrains are similar in this sense to most natural images. In such scenes, high gradient values or discontinuities are rare and most content changes gradually across the domain. Examining the spatial frequency domain of elevation maps shows that the lower spatial frequency components contains more energy than the high frequency components, which often correspond to details and noises. However, Psychophysical experiments suggest that humans are more receptive to losses in higher spatial frequency components than to losses in lower frequency components. Indeed, the quality of a general image compression method is often tested according to perception awareness. Moreover, image compression methods like *JPEG* and *JPEG2000* were originally designed to eliminate 'artifacts' in the compressed images. Although terrains can be efficiently compressed using DIP methods. Different error norms should be used, since terrain images represent a geometric surfaces (not just an images). One can think of several ways to measure the geometric quality of a simplified terrain, Ben-Moshe et al. [1] suggested a measure of quality based on preserving inter-point visibility. Other geometric measures may test the topological elements such as watershed and the water-flow. Methods for measuring topological and geometric approximation quality of compressed terrains are strongly motivated by GIS application, yet these measures are not mathematical norms and often depend on implementation parameters. Therefore, in this paper we only report on experiments testing the following standard error norms: MSE, MAE, RMS, PSNR.

3.1 The *ICTS* framework

The general Frame-work of the *ICTS* algorithm includes the following stages:

- **Preprocessing Stage:** converting the grid based terrain into a gray scale image; this step involves translating the elevation data (positive / negative values) into the gray scale value range.
- **Presetting Stage:** setting the compression parameters according to the geometric nature of the terrain. This stage is the main contribution of our algorithm, it allows us to compress a terrain image according to its geometric properties (e.g. water-flow), while standard compressions only consider it as a $2D$ natural image.
- **Compression stage:** perform the actual DIP compression. For most terrain-images the *DCT* compression is used.
- **Output testing stage:** The simplified terrain is compared to the original input. In case the output does not satisfy the user limitation (e.g. the error rate is too big) the compression parameters will be updated and a new compression will be computed. This stage is optional but might be needed in order to guarantee that the simplified terrain satisfies the user limitation.

3.2 The Parameters of Terrain Image Compression

Based on the well-known corollary that the values of the different parameters have a major effect on any DIP compression quality, we investigate the nature of the parameters which take part in

images-compression general framework, and demonstrate the role each parameter-value takes in the compression process. The following hi-level parameters should be fixed in order to perform a *DCT* compression: **Block Size**: usually $8 * 8$ pixels, but may also be of a general rectangle dimension (e.g. $8 * 16$ $16 * 16$ $32 * 32$). **Quantization Table**: takes an important roll in the trade-off between resolution and compressed-image size. **File Size**: limits the output terrain size, implies the compression ratio.

In order to tune the compression parameters to fit the specific nature terrains the pre-compression stage includes the following steps:

Input settings: the original terrain and its meta-data regarding both the input and the output type, size and other constrains. For instance, the user can ask to compress the terrain to a certain size. Alternatively, the user can ask to simplify the terrain to the minimal size given an upper error rate bound. The user can also suggest the type of the terrain.

Compute simple local parameters: In this part several local parameters are computed including: The min/max height (the extreme values of the terrain). The average height difference between a pixel and consecutive pixels (usually 8 neighbors). The Standard deviation of the difference between a pixel and consecutive pixels. This stage is highly efficient in terms of runtime and memory.

Compute global approximation factors: A rough approximation of the water flow and the watershed of the terrain, We followed known algorithms for computing watershed and water flow [2], A statistical representation of the water flow/shed can be computed using the same single 'pass' performed to compute the local parameters. Therefore this step is also implemented efficiently, since only a rough approximation of the water flow/shed is being computed (and not the complete diagrams).

Classify the terrain: Using the above parameters we classify the terrain into the following types: Flat or almost flat terrain. Mostly dunes (dunes have unique shape of water flow/shed). Hilly terrain and Smooth Mountains (i.e. old mountains), peaks and cliffs (i.e. new mountains). Natural terrain with artifacts (watershed exist: lakes, buildings). Natural terrain without a water flow (ocean surface terrains, stars surface terrains). Artificial terrains (i.e. cities, gaming...)

Set compression parameters: using the above computed information, we turn to set the compression parameters, including the block size, the proper quantization table, and other parameters.

4 Geometric Queries

In this section we discuss the usability of compressed terrains as geometric surfaces. Regular compressed images mostly represents general-2D images. In such case there is rarely a need of retrieving the value of a query 'pixel' without fetching all the image. On the other hand, simplified or compressed terrains represent geometric surfaces and thus should support local (point location like) queries. One of the most common queries performed on terrain is *point location*. In such query a 2D point $p(x, y)$ is given, and we are interested in computing the z -value (height) of p as implied by the terrain. Kirkpatrick [16] showed how a triangulation can support such query in $O(\log n)$ time with only a linear (space) overhead. Other queries may include two points Line Of Site (*LOS*), and general local-region queries.

Compressed terrain image can support the above geometric quires efficiently and with very little overhead storage. *Region Of Interest (ROI)* is a general image compression technique which support querying desired blocks of pixels, without fetching the entire image. *ROI* is supported in the

JPEG2000 standard and had been implemented on other image compression methods [20]. Using *ROI* approach all the above mentioned geometric queries can be supported efficiently. Moreover compressed terrain images can also support another geometric type of query; Said et al. [25] presented efficient image coding to access a pixel range using *DCT* compression. This way one can access only parts of the terrain within some elevation range.

5 Experimental Results

In this section we report on some of our experiments with *ICTS*, as well as comparisons with other terrain simplification methods *QSlim* [6], *Terra* [9] and *JPEG2000* [13]. We compared these terrain simplification methods using the following four measures (error norms): MSE (mean squared error), PSNR (peak signal-to-noise ratio), MAE (mean absolute error), RMS (root mean squared).

We have implemented a preliminary version of *ICTS* using the several open source tools (in particular we used *jj2000*[3]). Our implementation of triangulation was used to evaluate and sample the triangulation based simplifications (i.e. *QSlim*, *Terra*). The *ImageMagick*[11] application was used to verify the computed error norms for all simplification algorithms. All experiments were performed on the following platform: *Intel Pentium M 1.73Ghz, 512 MB RAM*. The current version of *QSlim* (2.1, 2004) was used, while for *Terra* and *GcTin* implementation, the original sources [9, 22, 23] were used.

We used a data set of 17 input terrains representing different and varied geographic regions, including: terrains representing dunes, hills, mountains, craters, ocean surfaces, and more. Each input terrain covers a rectangular area of $10 \times 10 - 100 \times 100 \text{ km}^2$, and consists of 1,000,000 - 16,000,000 vertices.

5.1 First Experiment: *JPEG2000* vs. *ICTS*

The first experiment we performed compared *ICTS* to *JPEG2000*. We wanted to test if the suggested new terrain simplification method can do better than *JPEG2000* (*JPEG2000* is a new image coding system that uses state-of-the-art compression techniques based on wavelet technology). Preliminary results showed that terrains compressed by *ICTS* have a better error rate and smaller file size than those compressed by *JPEG2000*. To experiment the advantages of *ICTS* over *JPEG2000* (with respect to terrain simplification) we forced the file size of the terrains compressed by *ICTS* to be at least 10% smaller than the corresponding *JPEG2000* files, see table 1. The major difference between the two methods is demonstrated in figure 4.

Norm	JPEG2000	ICTS
<i>Filesize(kb)</i>	198.27	168.78
<i>MSE</i>	2.9756	1.0481
<i>MAE</i>	1.1574	0.4251
<i>RMS</i>	1.6586	1.0525
<i>PSNR</i>	52.302	55.950

Table 1. *ICTS* vs. *JPEG2000* Terrain simplification results (error norms), based on average results over all the terrain data set.

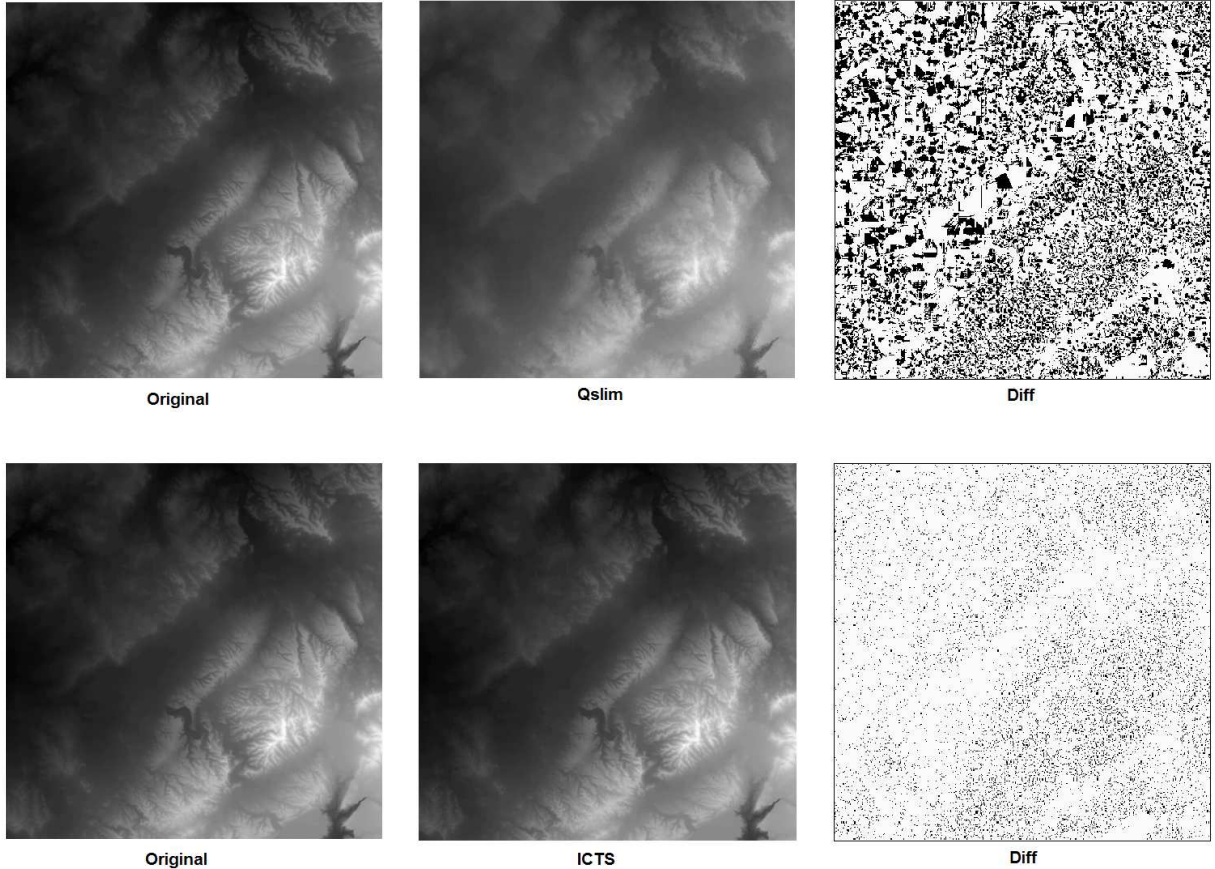


Fig. 1. The difference between a triangulation based simplification method and the image compression terrain simplification (using *DCT*) is illustrated.

5.2 Second Experiment *Terra* and *QSlim* vs. *ICTS*

AT the second experiment we compered between two types of terrain simplification methods: (i) standard; triangulation based terrain simplification (*QSlim*, *Terra*) and (ii) *ICTS* method.

In order to compare the compression quality (error norms) we first simplified the input terrains using *Terra* and *QSlim* into five levels of simplification: 10,000, 30,000, 50,000, 100,000, 200,000 (vertices). Then we compute the error norms for each simplified terrain. We then used *ICTS* to simplify each terrain to files of the same size as the corresponding *Terra* (or *QSlim*)³.

Table 2 shows the average error norms over all tested terrains and levels of simplification for the same file size compressed terrains.

Runtime results: For small terrains (elevation maps of 100,000 vertices), *ICTS* runs on average 4-10 times faster than *Terra* and *QSlim*. For larger terrains (1,000,000 vertices and more) the runtime gap grows significantly, often reaching a factor of 100. The larger the original terrain is the greater the gap between the methods (runtime) grows significantly. we were unable to run *QSlim*

³ To make sure the comparison is fair, each triangulation (simplified using *Terra* or *QSlim*) was farther compressed using standard *ZIP* compression. Only then the corresponding *ICTS* terrain was computed.

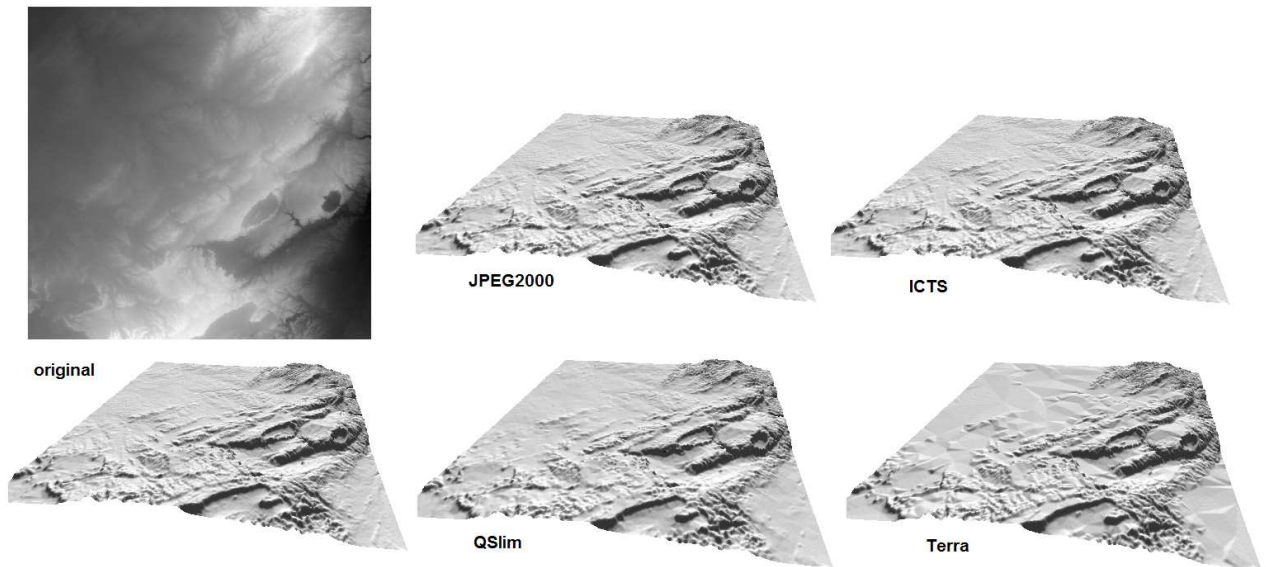


Fig. 2. The original elevation map compressed using all four simplification methods, and presented as simplified geometric surfaces.

Norm	QSlim	Terra	ICTS
<i>MSE</i>	54.9915	25.4917	0.4434
<i>MAE</i>	5.2899	3.7039	0.1725
<i>RMS</i>	7.06465	4.9786	0.6638
<i>PSNR</i>	39.7121	42.5078	59.9158

Table 2. *ICTS* vs. *QSlim* and *Terra* Terrain simplification results (error norms), based on average results over all the terrain data sets for the five levels of simplification.

or *Terra* because of memory limitations, while *JPEG2000* and *ICTS* compress such terrain in seconds (with minor memory overhead).

File size results: For the same error level the files computed *ICTS* were less than 15% of the corresponding same error ratio terrains simplified using *QSlim* or *Terra* and farther compressed by standard *ZIP*⁴. and then compressed using standard *ZIP* compression. Note: there are several compression methods designed specially for compressing triangulations [15]. Using these compressions might decrease the size of the compressed triangulation by factor of 1.5-2.5 comparing to standard *ZIP* compression, yet even with such compression these files will be at least 3 times larger than the terrains compressed by *ICTS*.

⁴ Compression methods designed specially for compressing triangulations [15] may decrease the size of the compressed triangulation by factor of 1.5-2.5 comparing to standard *ZIP* compression, yet even with such compression these files will be at least 3 times larger than the terrains compressed by *ICTS*.

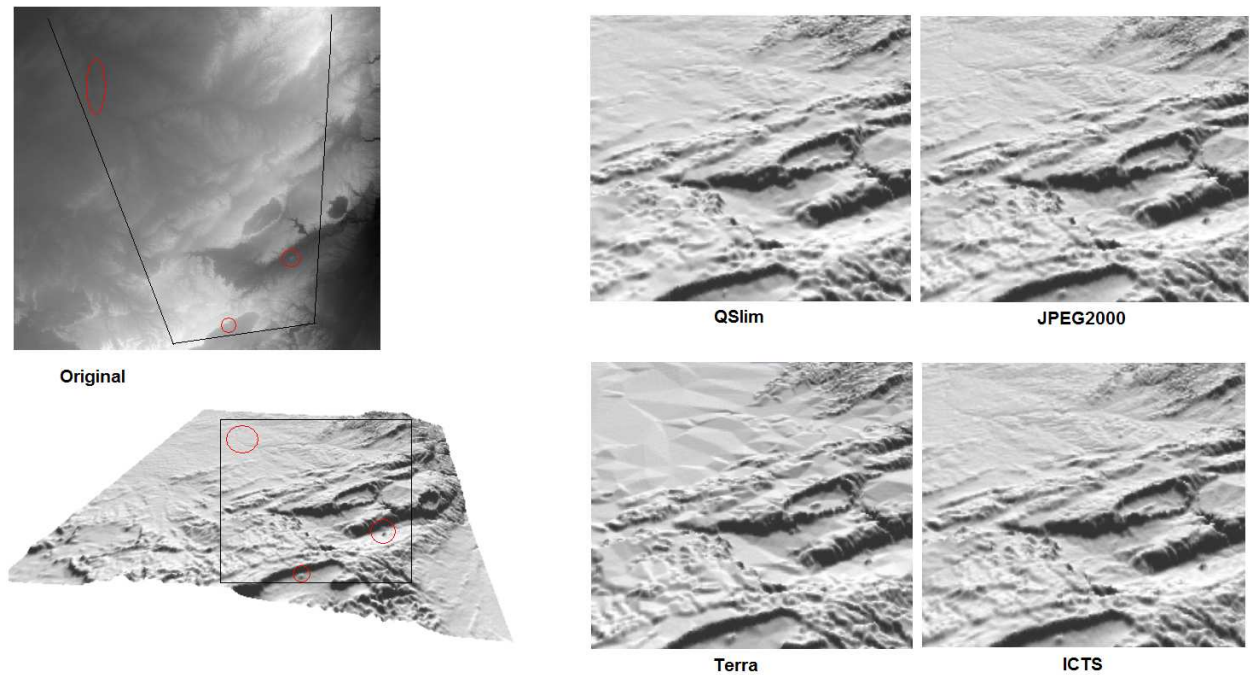


Fig. 3. The original 100*100 km terrain was simplified using all four methods (*QSlm*, *Terra*, *JPEG2000*, *ICTS*) all simplified terrains have (more or less) the same maximal vertical error, yet the *ICTS* terrain has the smallest file size.

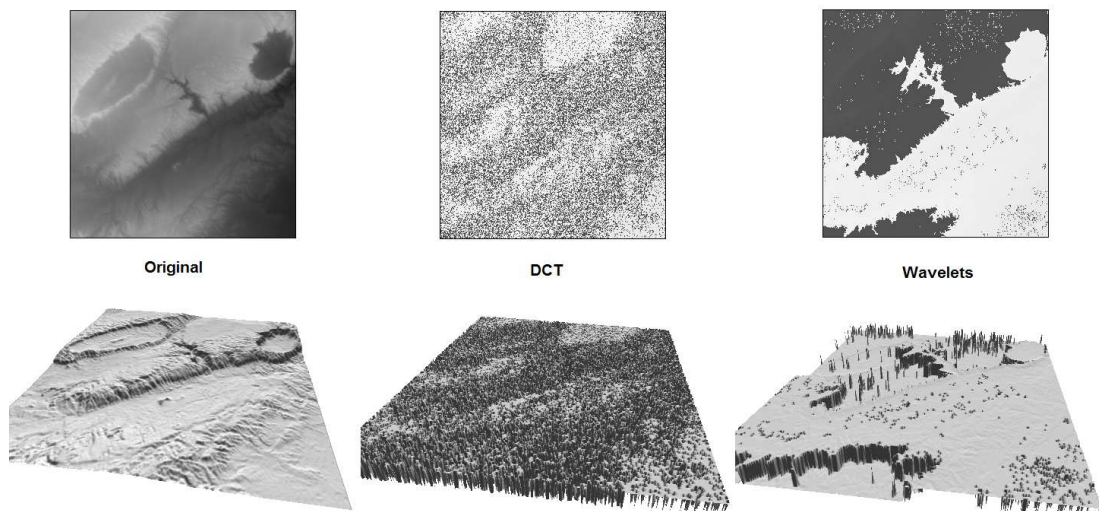


Fig. 4. The difference between *DCT* and wavelets (*DWT*) terrain-images compression is illustrated. The original terrain (on the left) was simplified using both *DCT* and *DWT*, the monochromatic images represent the error rate. A black dot presents an error rate larger than some constant (one meter), while a white dot implies that the approximated elevation value is within one meter distance to the original elevation. Observe that while the errors of the *DCT* compression seems almost as random points, the wavelet based compression divide the terrain into large regions of errors.

RMS value	QSlim	Terra	JPEG2000	ICTS
4.0	1082.1	1130.5	33.4	33.8
3.53	1418.6	1444.5	49.8	44.6
3.09	1839.5	1863.8	81.9	58.4
2.8	2113.1	2026.1	97.6	68.4
2.45	2163.2	2732.2	160.1	98.5

Table 3. The same error norm values(*RMS*) were used to test the file size (*kb*) of all simplification methods. For very large values of *RMS* *JPEG2000* compresses better than *ICTS*, and *Terra* compresses better than *QSlim*. For smaller *RMS* values *ICTS* performs better than *JPEG2000*.

6 Conclusion

We have tested several existing DIP formats (e.g. *JPEG*, *JPEG2000*) and shown that these formats often lead to significantly good compression of terrains. As shown in the experiment results above even a standard jpeg compression compresses terrains significantly better (smaller data, better quality, and faster runtime) than specialized terrain simplification methods such as *Terra*. Yet, because the DIP regular formats were originally designed for compressing images (usually natural images) and not terrains, their parameters and other fine details of the implementation can be specialized for terrain-images and therefore improved even further. Simplifying terrains using *ICTS*-like methods, can also support region of interest (*ROI*) queries, with very little over head. Thus, *GIS* queries such as line of site (*LOS*) can be performed on the compressed terrains. Yet another advantage of DIP like terrain simplification methods has to do with the actual implementation platform; DIP algorithms may be implemented on dedicated hardware (such as GPU [7]), therefore implementing *ICTS* in hardware is feasible and might lead to further runtime-improvement.

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