The Focus of Attention Problem Revisited

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Abstract

The Focus of Attention (FOA) Problem is, given a set of targets and a set of sensors in the plane, to track the targets with 'maximum possible accuracy'. The accuracy is measured in terms of the angle subtended by the sensor pairs at the assigned targets. In this paper, we consider a scenario in which we are given n targets on a line l and $2n$ sensors on a line m such that $l \,||\, m$ and the objective is to assign sensor-pairs to the targets such that the minimum angle subtended at the targets is maximized. We give a polynomial time algorithm for a restricted version of this problem and also study some properties of the optimal solution. To our knowledge, no deterministic and exact polynomial time algorithm is known for any non-trivial version of the FOA problem when the accuracy is measured in terms of the angles subtended by the sensor pairs.

Keywords: Target tracking, Focus of Attention problem

1 Introduction

The Focus of Attention problem is motivated from the problem of tracking targets using sensor networks which is mainly used for the purpose of surveillance and monitoring tasks [3]. A limitation these sensors have is that one sensor is not capable of tracking a target. The sensors for example can be cameras and they can be used to estimate the position of a target. In practice, at least two such sensors are required to estimate the position of a target and for many cases like pan-tilt-zoom cameras one sensor cannot be used to track more than one target. For range sensors, three are required to localize a target [1]. In this paper, we will only consider the case in which each target can be tracked exactly by 2 cameras. To estimate the quality of tracking a good metric is required. The angle subtended by a camera pair at a target plays a crucial role in tracking the targets [4]. To have minimum uncertainty in the position of the targets the best measure is to assign the cameras to the targets such that the deviation of the subtented angle at each of the targets from 90 degrees is somewhat low. In other words if all the subtended angles are 90 degrees then the position of the targets is estimated very accurately. It follows from this that for a good tracking of a target, the angle subtended at the target should not be very small. A natural way to make sure that the no angle is very small will be to maximize the minimum angle subtended at the targets. In [1] the cameras are assumed to be on a line and the error associated with an assignment of cameras c_i and c_j with target k is Z_k/l_{ij} where l_{ij} is the distance between the cameras and Z_k is the distance of the cameras from the line containing the cameras. The objective is to find an assignment that minimizes the total error. Intuitively, if we fix Z_k and the angle is small then the value of l_{ij} will be small and error will be large. Hence this metric tries to capture the angles via an approximation. So, the natural question that arises is, can we work directly with angles and design efficient algorithms that can output an assignment that minimizes the maximum deviation from 90 degrees? Unfortunately, this problem has been shown to be intractable in [2]. Gfeller et al [2] have recently shown that given a set of $2n$ cameras and n targets in the plane, it is NP-complete to decide whether there exists an assignment of cameras to track targets such that each subtended angle is 90 degrees. They also give approximation algorithms for maximizing the minimum angle and maximizing the sum of angles when cameras are placed on a line. An extensive survey and motivation on this problem is provided in [2].

To our knowledge, no deterministic and exact polynomial time algorithm is known for any non-trivial version of the FOA problem when the accuracy is measured in terms of the angles subtended by the camera pairs. In this paper we achieve such a result for a restricted version of the problem.

2 Problem Definition

Consider n targets present on a line l_1 and $2n$ cameras on a line l_2, where l_1 is parallel to l_2. Two cameras c_i and c_j are said to subtend an angle θ at a target t_k, if $\angle c_i t_k c_j$ is θ. In an **assignment**, a camera c can focus on exactly one target and a target is focussed by exactly two cameras. Moreover, each camera has to be used in an assignment. Consider the following version of the

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focus of attention problem:
We have to find an assignment of cameras to targets such that

- The minimum angle of all possible assignments is maximized (Parallel-MAX-MIN Problem).

Let LHS denote the set of first n cameras and RHS denote the set of last n cameras (ordered from left to right).

3 Parallel-MAX-MIN Problem

We start off by proving a fundamental fact regarding this optimization problem.

Lemma 1 *There always exists an optimal solution $\mathcal{O}$ of the Parallel-MAX-MIN problem such that in $\mathcal{O}$ there are no two camera pairings c_i, c_j and c_k, c_l such that $i < j < k < l$ where i, j, k, l denote the index of the cameras in the sorted order from left.*

Proof Let camera pairs (c_i, c_j) and (k, l) be assigned to t_p and t_q repectively in an optimal solution where $i < j < k < l$. If t_p is to the left of t_q then we can swap the assignment to get a new assignment with pairings (c_i, c_k, t_p) and (c_j, c_l, t_q) and in this new assignment the subtended angles for both the assignments increase, hence the minimum angle in the solution does not decrease. Similarly, if t_p is to the right of t_q then the pairings (c_i, c_k, t_q) and (c_j, c_l, t_p) will give us a solution which is optimal as well.
The following can be easily derived from the previous lemma.

Corollary 1 *There exists an optimal solution in which each camera from the set of first n cameras from left is paired with a camera in the set of last n cameras.*

Consider the following decision version of the problem:

Given an angle θ does there exist an assignment of cameras and targets such that all the subtended angles in the assignment are at least θ

It is clear that if we can solve this problem then we can solve the MAX-MIN problem just by doing a binary search on the n^3 possible values of θ. If we choose a camera pair, c_i from LHS and c_j from RHS then there exists a **validity interval** I such that for every point p in the interval the angle subtented by c_i and c_j at p is at least θ. This interval is defined by the intersection of a ball B with the line containing the cameras such that the boundary of this ball is that circle passing through i and j, for which the angle subtented by the chord ij at its center is 2θ, if θ is acute and $2\pi - 2\theta$, if θ is obtuse. So, the decision problem is equivalent to decide whether there exists an assignment of cameras and targets such that if target t_k is assigned to camera pair (i, j) then it lies inside the validity interval of (i, j).

3.1 Some Properties of the Intervals

We first introduce some notations on which the rest of the paper is based. We label the first n cameras from left to right as $a_1, a_2, \ldots a_n$, the last n cameras from left to right as $b_n, b_{n-1}, \ldots b_1$ and the n targets from left to right as $t_1, t_2, \ldots t_n$. We denote the validity interval corresponding to camera pairs (a_i, b_j) for angle θ by $I_\theta(a_i, b_j)$. The words 'before' and 'after' will corresponding to ordering from left to right. We can now state the following lemmas which are easy to prove:

Lemma 2 *If $I_\theta(a_i, b_j)$ and $I_\theta(a_k, b_l)$ are two intervals with $i \leq k$ and $j \leq l$, then $I_\theta(a_i, b_j)$ is nested within $I_\theta(a_k, b_l)$.*

Corollary 2 *If $I_\theta(a_i, b_j)$ covers a target t then it is also covered by $I_\theta(a_k, b_l)$ for all $k \leq i$ and $l \leq j$.*

3.2 Polynomial time Algorithm for a Restricted Case

In this section we describe the conditions that if imposed on the problem instance can bring some nice structure to the intervals defined by the cameras which can be exploited to get a greedy strategy work for the problem. We call this constraint **Interval Property** which can be stated in the following way:

For every camera $a_i \in LHS$, $I_\theta(a_i, b_n)$ should start before the interval $I_\theta(a_{i+1}, b_1)$ and for every camera $b_j \in RHS$, the interval $I_\theta(a_n, b_j)$ should end after the interval $I_\theta(a_1, b_{j+1})$.

Given an instance of the Parallel-MAX-MIN Problem, let Θ be the set of all angles θ such that there exist $a_i \in LHS, b_j \in RHS$ and target t_p with $\angle a_i t_p b_j = \theta$. We impose the following constraint C in order to make our algorithm work:
C: For each $\theta \in \Theta$ the following should hold
1. All the targets are to the left of the right end-point of the interval $I_\theta(a_n, b_n)$.
2. Interval Property is satisfied for each angle $\theta \in \Theta$.
Next, we derive a geometric constraint under which Interval Property is satisfied for a given value of θ.

3.2.1 The Height Condition

Consider the defining circles C_i and $C_i{}'$ corresponding to the intervals $I_\theta(a_i, b_n)$ and $I_\theta(a_{i+1}, b_1)$. Let these circles intersect at 2 points, the one which is above the line of the cameras let it be called X. Let the distance of this point from the line containing the cameras be denoted by H_{a_i}. In order to impose the above restriction on the intervals we would like to have the distance between the parallel lines to be less than H_{a_i}. So, we can calculate the critical height for each a_i and b_j and the height h_c we choose should satisfy the condition $h_c < \min\{\min_{a_i \in LHS} H_{a_i}, \min_{b_j \in RHS} H_{b_j}\}$. The following calculations are done assuming $\theta < 90^\circ$. These can be analogously done for obtuse and right angle as well.

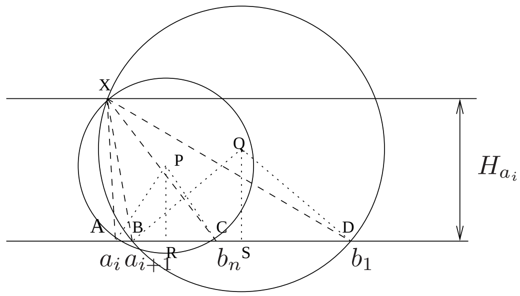

Figure 1: The Height Condition

In figure 1, $\angle AXC = \angle BXD = \theta$. Let the distance between a_i and b_n is $AC = d_1$, the distance between a_{i+1} and b_1 is $BD = d_3$ and $BC = d_2$. We use coordinate geometry to find the value of H_{a_i}. Let the center of C_i be P and that of C_i' be Q. It is clear that the radius of C_i is $r_1 = (d_1 + d_2)\text{cosec}\theta/2$ and that of C_i' is $r_2 = (d_2 + d_3)\text{cosec}\theta/2$. If the relative positions of the points A, B, C, R, S is as shown in the Figure 1 then the distance between the centres of the two circles is $RS = BC - SC - BR$. Now $SC = BC - BS = d_2 - (d_2 + d_3)/2 = (d_2 - d_3)/2$ and $BR = d_2 - (d_1 + d_2)/2 = (d_2 - d_1)/2$. Therefore $RS = (d_1 + d_3)/2$. The equation of C_i is $x^2 + y^2 = r_1^2$ (taking origin of coordinates at P) and that of C_i' is $(x - (d_1 + d_3)/2)^2 + (y - (d_3 - d_1)\cot\theta/2)^2 = r_2^2$. If we solve for the intersection of these circles we get

$$x^2 + y^2 - r_1^2 = (x - \frac{(d_1 + d_3)}{2})^2 + (y - \frac{(d_3 - d_1)\cot\theta}{2})^2 - r_2^2$$

$$r_1^2 - r_2^2 = \frac{(d_1 + d_2)^2}{4} + x(d_1 + d_3) + \frac{(d_3 - d_1)^2\cot^2\theta}{4} + y(d_3 - d_1)\cot\theta$$

Let,

$$\begin{aligned} A_i(\theta) &= \frac{(r_1^2 - r_2^2 - (d_1 + d_3)^2/4 - (d_3 - d_1)^2\cot^2\theta/4)}{(d_1 + d_3)} \\ B_i(\theta) &= \frac{(d_3 - d_1)\cot\theta}{(d_3 + d_1)} \end{aligned}$$

Then we have $x = A_i(\theta) + B_i(\theta)y$. Putting this value in the equation $x^2 + y^2 = r_1^2$ we get a quadratic equation in y whose positive root we denote by y_{pos}. Therefore, $H_{a_i} = y_{pos} + (d_1 + d_2)\cot\theta/2$. So, in order to force the aforementioned structure on the intervals we need to select $h \le h_c$.

Before going for the algorithm we state the following lemmas which are quite easy to see and hence we omit their proofs.

Lemma 3 *Given that interval property is satisfied, if interval $I_\theta(a_i, b_j)$ is nested by $I_\theta(a_k, b_l)$ then $k \le i$ and $l \le j$.*

Lemma 4 *Given that interval property is satified, if the left end-point of $I_\theta(a_i, b_j)$ is to the left of left end-point of $I_\theta(a_k, b_l)$ and right end-point of $I_\theta(a_i, b_j)$ is to the left of right end-point of $I_\theta(a_k, b_l)$ then $i < k$ and $l < j$.*

The arrangment of the intervals satisfying interval property ($n = 3$) for a particular instance is shown in Figure 2

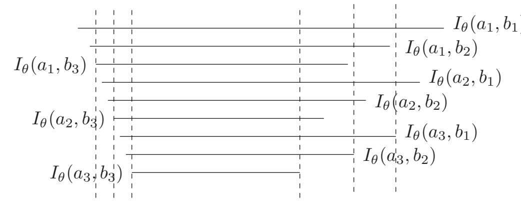

Figure 2: Arrangement of Intervals satisfying Interval Property

3.3 Algorithm

Now the question arises, how can this structure help us in getting a greedy strategy work. We assume a restriction on the where the cameras are placed which is as follows. We suggest a greedy strategy that works in this case. The strategy is as follows:

Consider the targets from left to right. For the i^{th} target from the left, let $I_1, I_2, \ldots I_m$ be the intervals in which it lies, assign it to the interval among these which is ending earliest. If the interval is $I_\theta(a_i, b_j)$ then remove all the intervals with left camera as a_i and right camera as b_j.

It can easily be seen that this algorithm can be implemented in $O(n^3 \log n)$ time.

Proof of Correctness of Algorithm

Proof. Let the given settings of cameras and targets have a solution as an assignment and let a valid assignment be called a **real assignment**. There can be many real assignments possible for an instance of the decesion problem. In this proof we will fix a real assignment (if there is any) and refer to that allthrough. A triple (a_i, b_j, t_p) is said to be a valid **pairing**, if the target t_p lies inside the interval defined by camera pair a_i, b_j. Consider the leftmost target t_1, and let $I_1, I_2, \ldots I_m$ be the intervals in which it lies. Also assume, $I_1 = I_\theta(a_i, b_j)$ be the interval that is finishing earliest among these. Suppose, in the real assignment, t_1 be assigned to the interval $I_2 = (a_k, b_l)$.

By case analysis we show that we can get a solution from the real solution which has the triple (a_i, b_j, t_1) in the assignment. So, we can now remove the triple (a_i, b_j, t_1) from the real assignment and the rest of the $n-1$ targets and $2(n-1)$ cameras will have a solution. The base case $(n = 2)$ can be verified easily. The following cases need to be handled:

Case 1. $j \neq n$. If I_1 is one of $I_\theta(a_1, b_1), I_\theta(a_1, b_2), \ldots, I_\theta(a_1, b_{n-1})$, say $I_\theta(a_1, b_j)$, $j \leq n-1$ then only intervals that can cover it are $I_\theta(a_1, b_1), I_\theta(a_1, b_2), \ldots, I_\theta(a_1, b_j)$. So, in the real assignment it must be covered by $I_\theta(a_1, b_1)$ where $k \leq j$. If $k = j$ we are done, else if $I_\theta(a_{i_1}, b_j)$ covers some target t_p then we can swap these to get new pairings as $(a_1, b_j, t_1), (a_{i_1}, b_k, t_p)$ and this will be a valid assignment because of above conditions.

Case 2. $j = n$. The following subcases arise:
(a) $k \leq i$. Let $k < i$, then because of the imposed constraint on the intervals, $I_\theta(a_k, b_l)$ will start before $I_\theta(a_i, b_j)$ and since it is ending after $I_\theta(a_i, b_j)$ it will be nesting the interval $I_\theta(a_i, b_j)$. Hence, from lemma 3 we have $j \leq l$. Now let in the real assignment we have the following 3 assignments $(a_k, b_l, t_1), (a_i, b_{j'}, t_p), (a_{i'}, b_j, t_q)$. We interchange the pairings to get 3 new pairings $(a_i, b_j, t_1), (a_k, b_{j'}, t_p), (a_{i'}, b_l, t_q)$ which are valid as $I_\theta(a_i, b_{j'})$ is nested in $I_\theta(a_k, b_{j'})$ and $I_\theta(a_{i'}, b_j)$ is nested in $I_\theta(a_{i'}, 1)$.

(b) $k > i$. If $k > i$, then starting point of $I_\theta(a_k, b_l)$ will be to the right of that of $I_\theta(a_i, b_j)$ and hence the intervals $I_\theta(a_i, b_j)$ and $I_\theta(a_k, b_l)$ satisfy the conditions of lemma 4, which implies $l < j = n$. Now assume that the real assignment has the triples $(a_k, b_l, t_1), (a_i, b_{j'}, t_p), (a_{i'}, b_j, t_q)$. Now, the new pairings we would like to propose are $(a_i, b_j, t_1), (a_k, b_{j'}, t_p), (a_{i'}, b_l, t_q)$. In this case the only problematic pair is $(a_i, b_{j'}, t_p), (a_k, b_{j'}, t_p)$ because $I_\theta(a_k, b_{j'})$ is nested in $I_\theta(a_i, b_{j'})$. If t_p lies in $I_\theta(a_k, b_{j'})$ then we are done. If not, because of the constraint on the position of cameras t_p can lie only to the left of $I_\theta(a_k, b_{j'})$. Since $j' < n = j$, $I_\theta(a_i, b_j)$ will be nested in $I_\theta(a_i, b_{j'})$. Also l cannot be greater than j' because if it is the case $I_\theta(a_k, b_l)$ will be nested in $I_\theta(a_k, b_{j'})$ and since t_1 is the leftmost camera t_p will lie to the right of it and hence will be covered by $I_\theta(a_k, b_l)$ as well, implying that it will be covered by $I_\theta(a_i, b_{j'})$ as well which is contrary to our assumption. Therefore, $l < j'$. Figure 3 shows the relative position of the intervals in this case. Hence in we can use the pairings $(a_i, b_j, t_1), (a_k, b_l, t_p), (a_{i'}, b_{j'}, t_q)$ to get a valid new solution. □

Therefore we can state the following theorem

Theorem 5 *Under the constraint C, the Parallel-MAX-MIN problem can be solved in $O(n^3 \log^2 n)$ time.*

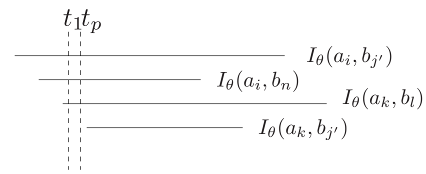

Figure 3: Case 3

Lemma 6 *If all the targets are placed to the left of the right end point of the interval $I_\theta(a_n, b_n)$ and the instance has a solution then there exists a valid assignment in which the leftmost camera is assigned to the leftmost target.*

Proof. In long version of the paper □

Using the above lemma we can prove a result for the case when targets can be placed arbitrarily on the line. For the following lemma assume that the given instance of the decision problem has a valid assignment.

Theorem 7 *When the cameras are positioned such that the interval property is satisfied then there exists a valid assignment in which the leftmost camera is assigned to the leftmost target or the rightmost camera is assigned to the rightmost target.*

Proof. In long version of the paper □

4 Conclusion

We study the Parallel-MAX-MIN version of the Focus Of Attention Problem and give a polynomial time algorithm assuming a constraint on the positioning of the cameras and the targets.

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