Rounding Face Lattices in the Plane

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Abstract

Recent theoretical work has shown that the exact coordinate representation of order types (collections of points and line segments) requires exponential storage (Goodman, Pollack, Sturmfels, 1988). This result supports the view that in any practical application of computational geometry, a certain amount of rounding of numerical quantities is required to bound the bit-complexity and storage requirements. Unfortunately, the naive use of rounded floating point arithmetic inevitably leads to unreliable programs. A theory of robust floating point geometry exists (Milenkovic, 1986, 1988), but it is not yet generally applicable.

One possible approach to the problem of attaining practical efficiency and reliability is to round at a "higher level". We describe here an algorithm for rounding a ***planar face lattice***, a decomposition of the plane into vertices, line segment edges, and polygonal regions. The algorithm can apply any order preserving rounding function R ($a \leq b$ implies $\mathrm{R}(a) \leq \mathrm{R}(b)$) to the vertex coordinates. Face lattices are common to many geometric algorithms, and by judicious use of lattice rounding, we can reduce the bit-complexity and storage requirement of these algorithms without sacrificing reliability.

A simple physical analogy illustrates the action of lattice rounding. Each vertex acts like a point of attachment for a set of elastic cords which represent the edges. As we round the vertex locations, the points of attachment drag their edge-cords along with them. Each attachment also deflects any other edge-cords it encounters. Under these constraints, each cord contracts to its minimum possible length. We prove that if the rounding is order preserving, the result is unique. The rounding algorithm is based on the shortest path algorithm for simple polygons (Guibas, Hershberger, Leven, Sharir, Tarjan, 1986). The cost of rounding edge **E** is $\mathrm{O}(V_{\mathbf{E}})$, where $V_{\mathbf{E}}$ is the number of vertices near to **E** (within the maximum vertex rounding distance). The total cost of determining which vertices lie near to each edge in the lattice is $\mathrm{O}(E \log E)$ where E is the number of edges.

The rounded value of an edge is not a straight line segment in general, but it is a polygonal path. This path is ***monotonic***: it does not "backtrack" with respect to the original line segment. Also, it stays within the maximum vertex rounding distance of the original segment. Thin portions of a polygonal region may be squeezed flat, and two edges may be forced to touch, but never cross, each other. New line segments are created only by joining existing vertices; no new vertex locations are introduced. It is shown that this type of approximation can be used to implement the solution to any problem arising from the physical domain. It is clearly superior to floating point techniques currently in use.

We show that the lattice rounding algorithm leads to provably reliable rounded arithmetic implementations for rotations, line segment intersections, polygon clipping, and many other operations.