The Visibility Graphs of Spiral Polygons

Hazel Everett

Dept. of Computer Science, University of Toronto, Toronto, Ontario, Canada, M5S 1A4.

Abstract

Two vertices of a polygon P are ***visible*** if the closed line segment between them does not intersect the exterior of P. The ***visibility graph***, G, of P is the graph whose vertices correspond to the vertices of P and two vertices are adjacent in G if the corresponding vertices in P are visible. The recognition problem for visibility graphs is, given a graph, determine whether it is the visibility graph of a simple polygon. No good upper bound on the complexity of this problem is known.

A ***spiral polygon*** is a simple polygon whose boundary chain contains exactly one concave subchain. I will show how to determine, in linear time, whether a graph is the visibility graph of a spiral polygon. A polygon is a ***k-spiral polygon*** if its boundary chain contains exactly k concave subchains. Every polygon is k-spiral for some k. The result presented here can be viewed as a solution to the 1-spiral case. Perhaps this solution will lend insight to the more general problem.

A fact that is used extensively in the recognition algorithm is that visibility graphs of spiral polygons are interval graphs. A graph is an *interval graph* if its vertices can be put into one to one correspondence with a set of intervals on the real line such that two vertices are connected by an edge iff their corresponding intervals have non-empty intersection. An important property of interval graphs is that their maximal cliques can be linearly ordered such that for every vertex v in the graph, the maximal cliques containing v occur consecutively [1]. Moreover, the maximal cliques can be listed in order in linear time [2]. In the visibility graph of a spiral polygon the maximal cliques are ordered from one end of the spiral to the other.

Not all interval graphs are visibility graphs of spiral polygons and the remaining necessary conditions will be presented. To show that these conditions are sufficient I will show how to construct, given a graph G satisfying the necessary conditions, a spiral polygon which has visibility graph G.

[1] P. C. Gilmore and A. J. Hoffman, A characterization of comparability graphs and of interval graphs, *Canad. J. Math.* **16** (1964), 539-548.

[2] K. S. Booth and G. S. Lueker, Testing for the consecutive ones property, interval graphs and graph planarity using PQ-tree algorithms, *J. Comput. System. Sci.* **13** (1976), 335-379.