Algorithms for Ham Sandwich Cuts

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Given disjoint sets $P_1, P_2, \ldots, P_d$ in R^d with n points in total, a *ham-sandwich cut* is a hyperplane that simultaneously bisects the P_i. Algorithms for finding ham-sandwich cuts in every dimension $d > 1$ will be described. When $d = 2$, the algorithm is optimal, having complexity $O(n)$. For dimension $d > 2$, the bound on the running time proportional to the worst-case time needed for constructing a level in an arrangement of n hyperplanes in dimension $d - 1$. This, in turn, is related to the number of k-sets in R^{d-1}. With the currently known estimates, the complexity is close to $O(n^{3/2})$ for $d = 3$, roughly $O(n^{8/3})$ for $d = 4$ and $O(n^{d-1-a(d)})$ for some $a(d) > 0$ (going to zero as d increases) for larger d. It is surprising that the complexity of finding a ham-sandwich cut is linear in R^3 and R^4 if the sets are suitably separated.